Real Numbers

- **Introduction**
  - Real numbers include all numbers on the number line
  - They consist of rational numbers and irrational numbers
  - Rational numbers: integers, fractions, decimals (e.g., 7, -3, 0.5, 4/3)
  - Irrational numbers: cannot be written as p/q, non-terminating non-repeating decimals (e.g., $\sqrt{2}$, π)
- **Classification of Real Numbers**
  - Natural Numbers (N): Counting numbers {1, 2, 3, ...}
  - Whole Numbers (W): Natural numbers plus zero {0, 1, 2, 3, ...}
  - Integers (Z): Whole numbers and their negatives {..., -3, -2, -1, 0, 1, 2, 3, ...}
  - Rational Numbers: Numbers in the form p/q, where p and q are integers, $q \neq 0$ (e.g., 8/11, -3/17)
  - Irrational Numbers: Numbers not expressible as p/q, such as $\sqrt{5}$, $\sqrt{3}$, π
- **Fundamental Theorem of Arithmetic**
  - Composite numbers have factors other than 1 and itself (e.g., 10: factors 1, 2, 5, 10)
  - Prime numbers have exactly two factors: 1 and itself (e.g., 23)
  - Every composite number can be uniquely expressed as a product of prime factors (order may vary)
  - Example: $270 = 2 \times 3^3 \times 5$
- **HCF and LCM by Prime Factorization**
  - For two positive integers a and b: $HCF(a, b) \times LCM(a, b) = a \times b$
  - HCF: product of smallest powers of common prime factors
  - LCM: product of highest powers of all prime factors involved
  - Example:
    - $120 = 2^3 \times 3 \times 5$
    - $144 = 2^4 \times 3^2$
    - $HCF(120, 144) = 2^3 \times 3 = 24$
    - $LCM(120, 144) = 2^4 \times 3^2 \times 5 = 720$
  - For $p = a \times b^2$ and $q = a^3 \times b$, $LCM(p, q) = a^3 \times b^2$
- **Applications of HCF and LCM**
  - Smallest odd composite number (9) and smallest odd prime (3):
    - HCF = 3, LCM = 9
  - Problem involving HCF and LCM:
    - Given HCF(253, 440) = 11 and LCM(253, 440) = $253 \times R$, find R
    - R = 40
  - Real-life example:
    - Ravi takes 16 min and Shikha takes 20 min to complete a round
    - They meet again after LCM(16, 20) = 80 minutes
- **Revisiting Irrational Numbers**
  - Irrational numbers cannot be expressed as p/q
  - Square roots of prime numbers are irrational (e.g., $\sqrt{2}$, $\sqrt{3}$)
  - Square roots of some numbers are rational (e.g., $\sqrt{4} = 2$)
  - Theorem: If a prime p divides a^2, then p divides a
  - Proofs:
    - $\sqrt{2}$ is irrational
    - $3\sqrt{2}$ is irrational
    - $5 - \sqrt{3}$ is irrational